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NUMERICAL ANALYSIS OF DEFORMATIONS OF THE RECTANGULAR (SQUARE AND STRIP) SHALLOW FOUNDATIONS

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1. Introduction

In this contribution we shall deal with numerical analysis of the vertical deformation (settlement), deflection and relative deformation (relative settlement, relative deflection, flexibility) of rigid and flexible rectangular (square and strip) shallow foundations. Presented analysis is oriented on the effect of stiffness of system "foundation – subsoil" and bond (bi-directional bond and one-directional bond with and without friction) on the values of the deformations. In the foundations design and assessment according to the limit states we have to know their deformations. Generally, we expect that the deformations depend on the relative stiffness of the foundation in contact with subsoil. To the most significant factors affecting the relative stiffness of foundation belong:

- geometrical shape, dimensions and deformation properties of the foundations,
- non-homogenity, anisotropy and deformation properties of the subsoil,
- bond and friction on the contact area between foundation and subsoil,
- type, intensity and distribution of loading.

These factors significantly influence the determination of the input data in the numerical calculations and pre-determinate complexity of the boundary conditions of the solved problem. From the mathematical point of view, this problem is solved by deformation variant of the Finite Element Method (FEM). The numerically obtained results are presented in graphical and tabular forms. Results are qualitatively and quantitatively analyzed. Complex numerical analyses of the interaction of shallow (circular, square and strip) foundations with subsoil were published in [1].

2. Boundary conditions

The problem of the interaction between rectangular (square and strip) foundations and subsoil is solved by mathematical modelling using FEM. Computer program ANSYS® [4] was used to solve the problem. In the solution of the problem it was important to observe the physical principles. The contact task is solved as a 3-D problem according to assumptions of the linear elastic half-space theory.

Geometrical, material and static boundary conditions used in numerical calculations are described in detail in [2]. Brief description of the chosen boundary conditions (taken from [2]) is as follows:

a) Geometrical shape and stiffness of foundation structure

For solved strip foundation, rate of length "L" and width "B" is $L/B \cong 10$. The foundation relative stiffness "k" is defined according to the formula [3]:

$$k = \frac{E_f}{E_{def}} \left(\frac{t}{L} \right)^3 \qquad (1)$$

where "E_f" is the modulus of elasticity of a foundation and "E_{def}" is the modulus of elasticity of subsoil. For assessments of stiffness in width direction L=B. For relative stiffness k<1 the foundation is considered as flexible and for k>1 the foundation is considered as a rigid. Geometrical characteristics and stiffness of rectangular (square and strip) shallow foundations are listed in Tab. 1.

Table 1. Geometrical characteristics and stiffness of rectangular shallow foundations

Geometrical shapes of foundations	Ratio L/B (-)	Foundation dimensions			Foundation relative stiffness STN 73 1001 [3] k (-)
		Width B (mm)	Length L (mm)	Thickness t (mm)	
SQUARE	1	200	200	2.5 to 100	0.0159 - 1009.62
STRIP	≅ 10	65	630	5 to 100	0.004 - 32.3 *

* Stiffness in length direction "L" of strip foundations

b) Physical properties of the foundation and subsoil models

The physical properties of foundations and subsoil are listed in Tab. 2.

Table 2. Physical properties of foundations and subsoil

Model	Material	Physical properties		
		Modulus of elasticity E (MPa)	Poisson's ratio ν (-)	Relative density I_D (-)
Foundation	Steel	210 000	0.20	-
Subsoil	Sand	26	0.28	0.7

c) Bond and friction at the contact surface

From point of view of the effects of bond and friction, on the contact surface between the foundation and subsoil, three following cases were modeled:

- bi-directional bond (transmission of pressure and tensile forces, and shear forces at the solid contact between foundation and subsoil),
- one-directional bond with friction (transmission only due to pressure forces, and shear forces depended on the value of the angle of internal friction ϕ=35°),
- one-directional bond frictionless (transmission only due to pressure forces, and shear forces depended on the value of the angle of internal friction ϕ=0°).

The disadvantage of the bi-directional bond model is the transmission of the tension forces between foundation and subsoil.

d) Mathematical methods and calculation models

Soil-structure interaction is solved using the deformation variant of FEM. Three-dimensional finite element “SOLID45” is used for meshing continuous region of the foundation and subsoil model. Bi-directional bond is modeled using ”SOLID45” element between foundation and subsoil. One-directional bond are modeled using “CONTA174” and “TARGE170” contact elements. The Coulomb theory [4] for friction modeling between foundation and subsoil is used.

3. Evaluation of numerical results

From the numerical analyses of rigid and flexible rectangular (square and strip) shallow foundations a lot of qualitative and quantitative information about the effects of stiffness, bond (bi-directional, one-directional) and friction on the vertical deformations (settlements), deflections and relative deformations (relative settlement, relative deflection, flexibility) are obtained. Deformations were evaluated for representative points (axis, boundary and corner) of rectangular foundations, position of which is given in Fig. 3.

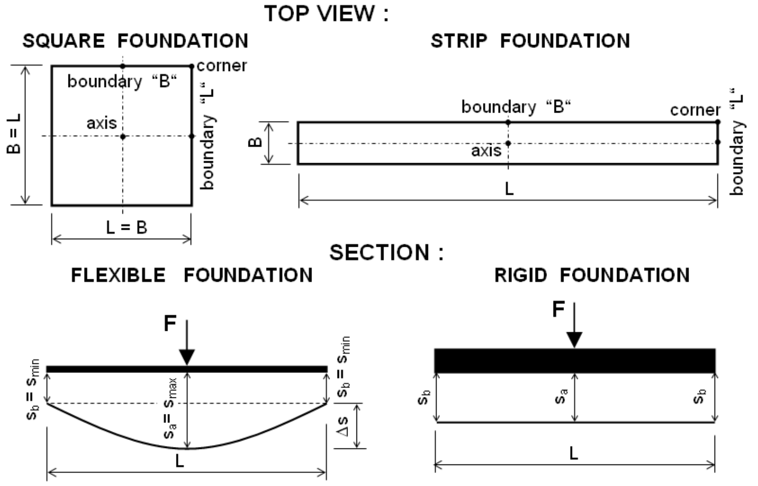

Fig. 3 Position of representative points on rectangular (square and strip) foundations and designation of vertical displacement (settlement) calculated for rigid and flexible foundations

From the calculated vertical displacements (settlements) of foundation models calculated by FEM in representative points (Fig. 3) followed relative characteristics were evaluated:

a) deflection calculated by formula: $\Delta s = s_{\max} - s_{\min}$ (2)

b) relative settlement for boundary of foundation calculated by formula: $s_b / s_{\max}$ (3)

c) relative deflection in length direction calculated by formula: $\Delta s / L$ (4)

d) flexibility calculated by formula: $\Delta s / s_{\max}$ (5)

Graphical interpretation of the stiffness, bond and friction effects on the size of deflection, relative settlement, relative deflection and flexibility for rectangular foundations (for boundary points in length direction) are shown in Figs. 4÷7. Isosurfaces of vertical displacements (settlements) calculated for rigid and flexible rectangular (square and strip) foundations (with bi-directional and one-directional bond with friction) for average contact stress σ_m=50kPa are shown in Figs. 8 and 9.

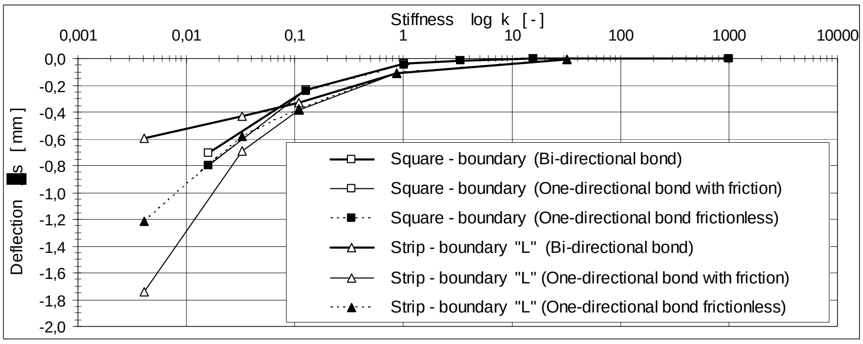

Fig. 4 Effects of stiffness, bond and friction on the deflection in boundary points (for strip foundation the boundary point is in length direction)

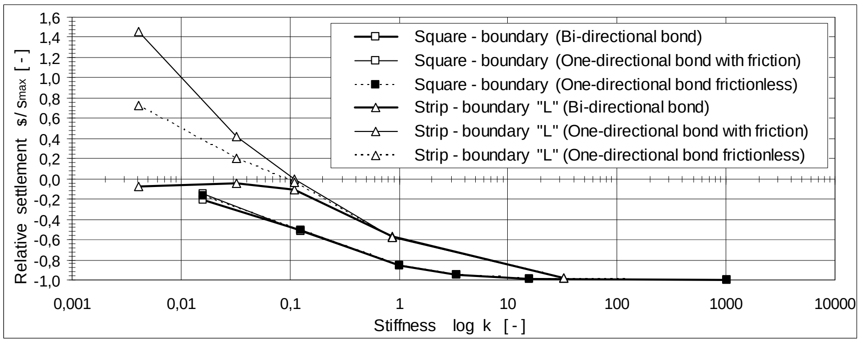

Fig. 5 Effects of stiffness, bond and friction on the relative settlement in boundary points (for strip foundation evaluated for boundary point in length direction)

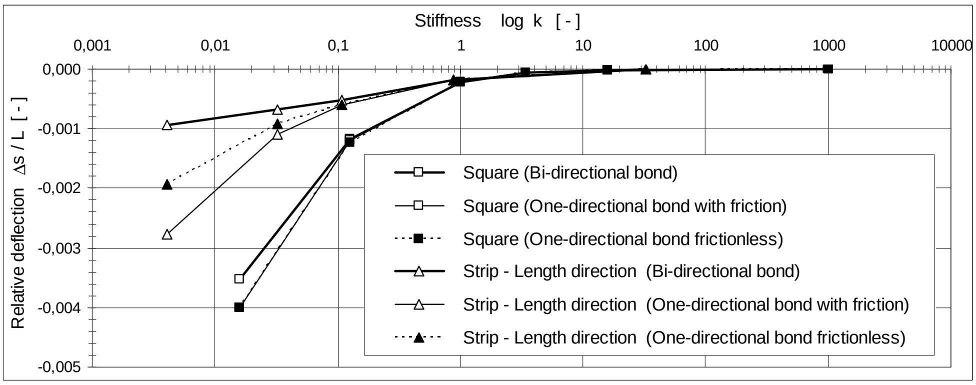

Fig. 6 Effects of stiffness, bond and friction on the relative deflection of rectangular foundations (for strip foundation evaluated in length direction)

Form the presented results we can see that effect of relative stiffness on deformations (deflection, relative settlement, relative deflection, flexibility) of square and strip foundations is very significant (in practical calculations non-negligible). The same value of relative stiffness "k" determined according to Eq. (1), the deformations of square foundations are less than with strip foundations, it means that square foundations are “shapely” stiffer. Square and strip foundations may be considered as perfectly rigid for the relative stiffness k>10. Bond and friction effects are for square foundation relative low. In the case of strip foundations the bond and friction effects are significant (in practical calculations non-negligible) when relative stiffness k<0.1.

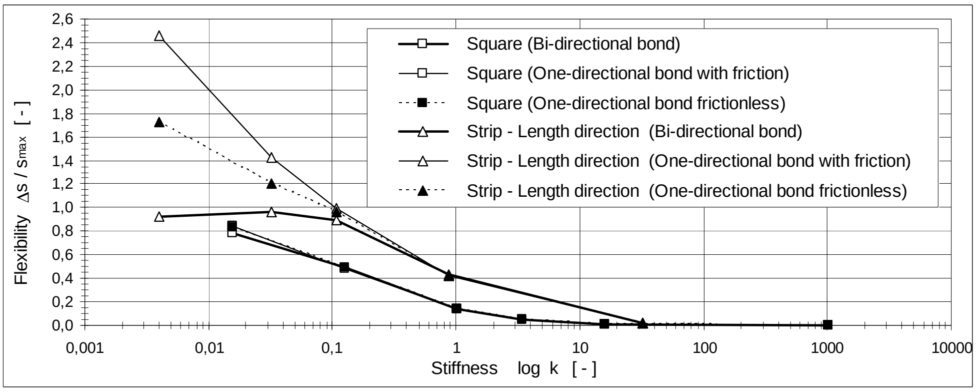

Fig. 7 Effects of stiffness, bond and friction on the flexibility of rectangular foundations (for strip foundation evaluated in length direction)

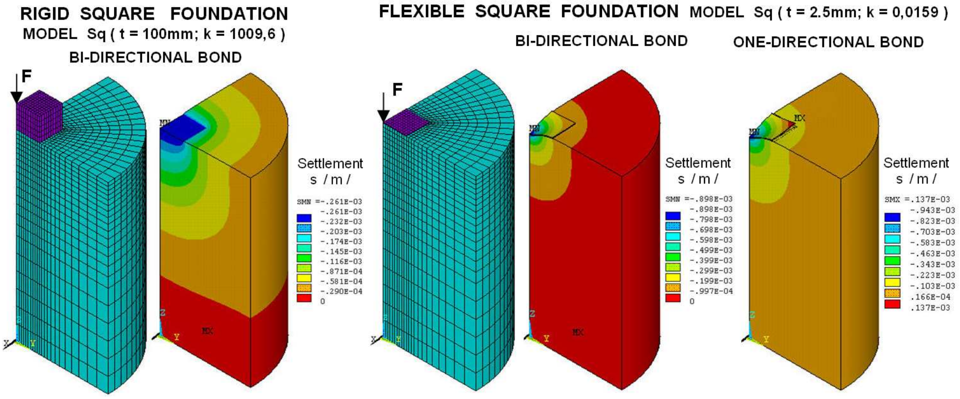

Fig. 8 Isosurfaces of settlements calculated for rigid (t=100 mm; k=1009.6) and flexible (t=2.5mm; k=0,0159) square foundations (with bi-directional and one-directional bond with friction) for average contact stress σ_m = 50 kPa

4. Conclusion

The application of FEM modeling allows to determine correctly the size of the vertical deformation (settlement), deflection and relative deformation (relative settlement, relative deflection, flexibility) of rigid and flexible rectangular (square and strip) shallow foundations. Presented calculations show that taking into account the effects of stiffness, bond and friction in the foundation bottom allows in practical solutions more realistic and economical design of rectangular foundations with respect of the required reliability.

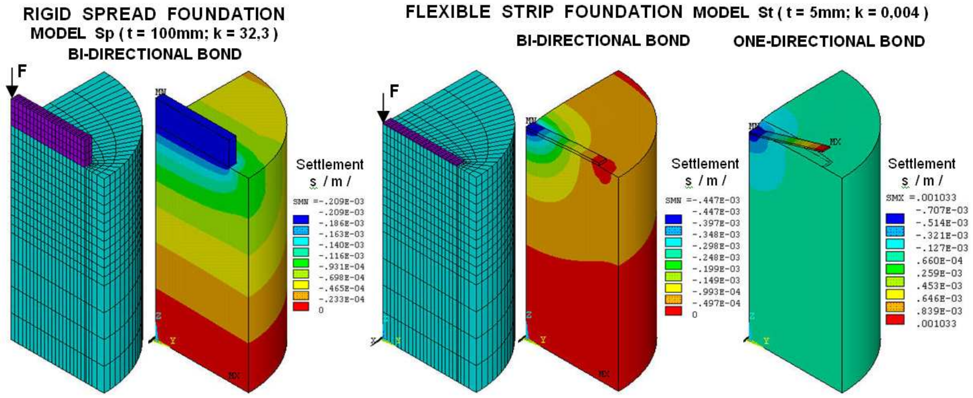

Fig. 9 Isosurfaces of settlements calculated for rigid (t=100 mm; k=32.3) and flexible (t=5mm; k=0,004) strip foundations (with bi-directional and one-directional bond with friction) for average contact stress σ_m = 50 kPa

References

[1] Hruštinec Ľ.: Interaction analysis of the shallow foundations with subsoil. Ph.D. Thesis, Bratislava, 2002, 689 pp. (In Slovak).

[2] Hruštinec, Ľ., Kuzma J., Sumec, J.: Numerical analysis of contact stresses under rectangular (square and strip) shallow foundations. Roczniki Inżynierii Budowlanej, Zeszyt 10/2010, p. 6.

[3] STN 73 1001: Subsoil under shallow foundation. Slovak Standard, 1987, (In Slovak).

[4] ANSYS®: User’s Manual, Swanson Analysis System, Inc., Volume I÷IV, 1999.

NUMERICKÁ ANALÝZA DEFORMÁCIÍ PRAVOUHLÝCH (ŠTVORCOVÝCH A PÁSOVÝCH) PLOŠNÝCH ZÁKLADOV

Zhrnutie

V článku sa zaoberáme numerickou analýzou deformácií tuhých a ohybných pravouhlých (štvorcových a pásových) plošných základov. Modelovanie pomocou MKP nám umožnilo výstižnejšie určiť veľkosť zvislej deformácie (sadnutia), priehybu a pretvorenie (pomerné sadnutie, relatívny priehyb a ohybnosť) základovej konštrukcie. Výsledky výpočtov preukázali, že pri zohľadnení vplyvu tuhosti, väzby a trenia na kontaktnej ploche základu s podložím dosiahneme reálnejší a hospodárnejší návrh pravouhlých plošných základov pri zachovaní požadovanej spoľahlivosti.

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