

NUMERICAL ANALYSIS OF CONTACT STRESSES UNDER RECTANGULAR (SQUARE AND STRIP) SHALLOW FOUNDATIONS

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1. Introduction

In this contribution we shall deal with numerical analysis of contact stresses under rigid and flexible rectangular (square and strip) shallow foundations. Presented analysis is oriented on the effects of stiffness of system “foundation – subsoil” and of bond (bi-directional and one-directional bonds with and without friction) on the value of vertical normal and shear contact stresses. In the foundations design and assessment, according to limit states we must know the intensity and distribution of contact stresses in the foundation bottom. A lot of various factors affect the distribution of contact stresses at the level of foundation bottom. Generally, we expect that the stress distribution depends on the relative stiffness of the foundation in connection with subsoil. To the most significant factors affecting the relative stiffness of foundation belongs:

- geometrical shape, dimensions and deformation properties of the foundations,
- non-homogeneity, anisotropy and deformation properties of the subsoil,
- bond and friction on the contact area between foundation and subsoil.

These factors significantly influence the determination of the input data in the numerical calculations and pre-determinate complexity of the boundary condition of the solved problem. Complex numerical analysis of the interaction of shallow (square, strip and circular) foundations with subsoil was published in [1]. Similarly, effects of foundation stiffness and bond on the contact surface on the settlement and the deformation of rectangular foundation are discussed in detail in [2].

2. Boundary conditions of the problem

The problem of the interaction between shallow foundations and subsoil is solved by mathematical modelling using Finite Element Method (FEM). Computer program ANSYS® [4] was used to solve the problem. In the solution of the problem it was important to observe the physical principles. The contact task is solved as a 3-D problem according to assumptions of the linear elastic half-space theory.

a) Geometrical shape and stiffness of foundation structure

Geometrical shape is one of the most general factors which affect the relative stiffness of the system. This factor can be relatively exactly determined for all shallow

foundations. Geometrical parameters and stiffness of solved square and strip foundation models are given in Tab. 1. For solved strip foundation, rate of length “L” and width “B”: is $L/B \cong 10$. The foundation relative stiffness “k” is defined according to the formula [3]:

$$k = \frac{E_f}{E_{def}} \left(\frac{t}{L} \right)^3 \quad (1)$$

where “ E_f ” is the modulus of elasticity of a foundation and “ E_{def} ” is the modulus of elasticity of subsoil. For assessments of stiffness in width direction $L=B$. For relative stiffness $k < 1$ the foundation is considered as flexible and for $k > 1$ the foundation it is considered as a rigid. In the case of square foundations, six models were solved with different stiffness “k” ($k=0.016$; 0.126 ; 1.009 ; 3.407 ; 15.78 ; 1009.62) depended of the various thickness “t” ($t=2.5$; 5 ; 10 ; 15 ; 30 ; 100mm). On the other hand, in the strip foundations five models were solved with different thickness “t” ($t=5$; 10 ; 15 ; 30 ; 100mm) and stiffness “k” ($k=0.004$; 0.032 ; 0.109 ; 0.872 ; 32.3), respectively.

Table 1. Geometrical characteristics and stiffness of rectangular shallow foundations

Geometrical shapes of foundations	Ratio L/B (-)	Foundation dimensions			Foundation relative stiffness STN 73 1001 [3] k (-)
		Width B (mm)	Length L (mm)	Thickness t (mm)	
SQUARE	1	200	200	2.5 to 100	0.0159 - 1009.62
STRIP	$\cong 10$	65	630	5 to 100	0.004 - 32.3 *

* stiffness in length direction “L” of strip foundations

b) Physical properties of the foundation and subsoil models

From the physical point of view, the model with steel foundation is considered. This model is put on the dense sand subsoil (Fig. 1). The volume of subsoil under foundation is modelled as the cylinder with diameter and height 0.8m. The physical properties of foundations and subsoil are listed in Tab. 2. Model of foundation was loaded by axial forces of various values “ F_i ” (axial forces F_1 to F_7), which generate the average contact stress of value from $\sigma_{m1}=3.2\text{kPa}$ to $\sigma_{m7}=76.5\text{kPa}$, i.e. in the elastic zone of dense sand subsoil.

Table 2. Physical properties of foundations and subsoil

Model	Material	Physical properties		
		Modulus of elasticity E (MPa)	Poisson's ratio ν (-)	Relative density I_D (-)
Foundation	Steel	210 000	0.20	-
Subsoil	Sand	26	0.28	0.7

c) Bond and friction at the contact surface

From point of view of the effects of bond and friction, to the contact surface between the foundation and subsoil, three following cases were modeled:

- bi-directional bond (transmission of pressure and tensile forces, and shear forces at the solid contact between foundation and subsoil),
- one-directional bond with friction (transmission only due to pressure forces, and shear forces depended on the value of the angle of internal friction $\phi=35^\circ$),

- one-directional bond frictionless (transmission only due to pressure forces, and shear forces depended on the value of the angle of internal friction $\phi=0^\circ$).

The disadvantage of the bi-directional bond model is the transmission of the tension forces between foundation and subsoil. Model with one-directional bond describes more precisely the behavior of real foundation.

d) Mathematical methods and calculation models

Soil-structure interaction is solved using the deformation variant of FEM. Three-dimensional finite element "SOLID45" is used for meshing continuous region of the foundation and subsoil model. Bi-directional bond is modeled using "SOLID45" element between foundation and subsoil. One-directional bond are modeled using "CONTA174" and "TARGE170" contact elements. The Coulomb theory [4] for friction modeling between foundation and subsoil is used. Schematic representation of the computing model of rigid square foundation ($t=100$ mm) with a bi-directional bond with resulting meshing of finite elements and the static boundary conditions is showed in Fig. 1. From the chosen boundary conditions it follows, that solved problem is axially symmetric. For this reason, the calculations have been performed on one-quarter of full calculation model.

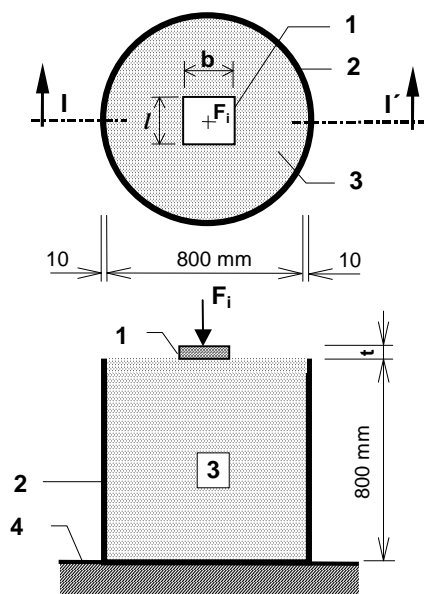


Fig. 1 Scheme of physical model of a square foundation and subsoil (1– foundation model; 2 – steel cylinder; 3 – sand subsoil; 4 – solid background)

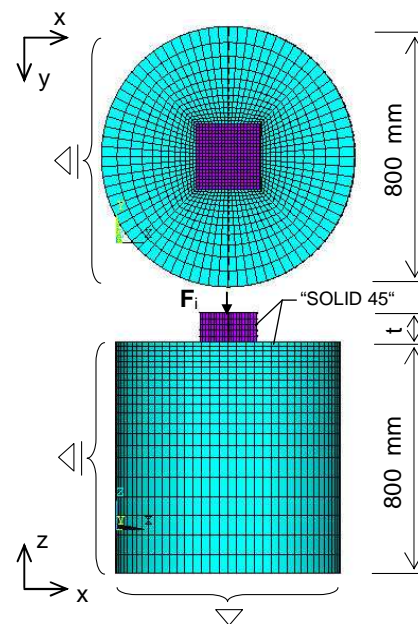


Fig. 2 Numerical models of rigid square foundation ($t=100$ mm) with bi-directional bond and static boundary conditions

3. Evaluation of numerical results

From the numerical results a lot of qualitative and quantitative information about the effects of stiffness of the shallow foundations and bond at the contact surface on intensity and distribution of vertical normal and shear contact stresses are obtained. In Tabs. 3 and 4 the final values of relative vertical normal and shear contact stresses in the representative

points (under axis, boundary and corner) for rigid ($k \gg 1$; or $k > 30$) and flexible ($k = 0,1$) foundation models are presented.

Table 3. Effects of stiffness, bond and friction on the value of relative vertical normal contact stresses in the foundation representative points

Geometrical shapes of foundations	Point location on contact surface	Vertical normal contact stresses σ_z/σ_m calculated under average loading intensity $\sigma_m = 50,0$ kPa								
		Bi-directional bond (with solid contact)			One-directional bond (without friction if $\phi=0^\circ$)			One-directional bond (with friction if $\phi=35^\circ$)		
		σ_z / σ_m [-]			σ_z / σ_m [-]			σ_z / σ_m [-]		
		R ($k \gg 1$)	R - F ($k=1,0$)	F ($k=0,1$)	R ($k \gg 1$)	R - F ($k=1,0$)	F ($k=0,1$)	R ($k \gg 1$)	R - F ($k=1,0$)	F ($k=0,1$)
Square	centre	-0,489	-0,868	-3,788	-0,487	-0,834	-3,101	-0,502	-0,869	-3,785
	boundary	-1,480	-1,367	-0,553	-1,412	-1,356	-0,786	-1,399	-1,387	-0,630
	corner	-3,095	-2,240	-0,515	-2,591	-1,956	-0,039	-2,500	-1,853	-0,022
Strip ($L/B=10$)	centre	-0,507	-0,702	-1,244	-0,562	-0,766	-1,315	-0,550	-0,752	-1,361
	boundary (L)	-1,035	-0,506	0,449	-1,006	-0,527	-0,028	-0,989	-0,516	-0,003
	corner	-1,438	-0,739	0,538	-1,068	-0,585	-0,014	-1,048	-0,568	-0,003

Remark: R – rigid foundation ($k > 1$); F – flexible foundation ($k < 1,0$);

R – F : stiffness on the boundary of rigid and flexible foundation ($k=1,0$)

Table 4. Effects of stiffness, bond and friction on the value of relative shear contact stresses in the foundation representative points

Geometrical shapes of foundations	Point location on contact surface	Shear contact stresses τ_{yz}/σ_m calculated under average loading intensity $\sigma_m = 50,0$ kPa								
		Bi-directional bond (with solid contact)			One-directional bond (without friction if $\phi=0^\circ$)			One-directional bond (with friction if $\phi=35^\circ$)		
		σ_z / σ_m [-]			σ_z / σ_m [-]			σ_z / σ_m [-]		
		R ($k \gg 1$)	R - F ($k=1,0$)	F ($k=0,1$)	R ($k \gg 1$)	R - F ($k=1,0$)	F ($k=0,1$)	R ($k \gg 1$)	R - F ($k=1,0$)	F ($k=0,1$)
Square	centre	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
	boundary	0,338	0,362	0,390	0,191	0,193	0,171	0,283	0,281	0,250
	corner	0,569	0,471	0,103	0,379	0,293	0,006	0,437	0,338	0,032
Strip ($L/B=10$)	centre	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
	boundary (L)	0,226	0,166	0,016	0,201	0,125	-0,010	0,225	0,162	0,007
	corner	0,267	0,200	0,030	0,152	0,107	-0,001	0,172	0,136	0,007

In the Figs. 3 and 4 the distribution of relative vertical normal contact stresses (σ_z/σ_m) under action of one-directional and bi-directional bond is shown. Similarly, in the Figs. 5 and 6 the distribution of relative shear contact stresses (τ_{yz}/σ_m) is presented. Relative stresses are evaluated under action of average normal stress $\sigma_m=50,0$ kPa. From the obtained results we can observe the significant effect, especially of stiffness and bond in foundation bottom on size and distribution of vertical normal and shear contact stresses. From the point of view of distribution of contact stresses, it is possible to consider square and strip foundation as ideally rigid for relative stiffness $k > 10$.

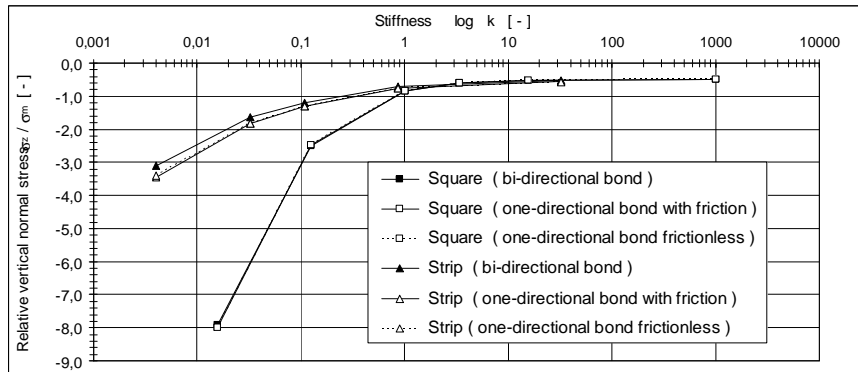


Fig. 3 Effect of stiffness, bond and friction on the values of relative vertical normal contact stresses (σ_z / σ_m) under axis of foundations

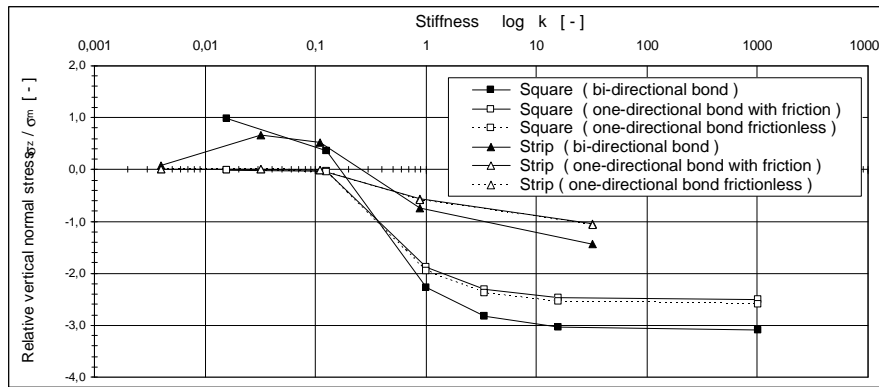


Fig. 4 Effect of stiffness, bond and friction on the values of relative vertical normal contact stresses (σ_z / σ_m) under corner of foundations

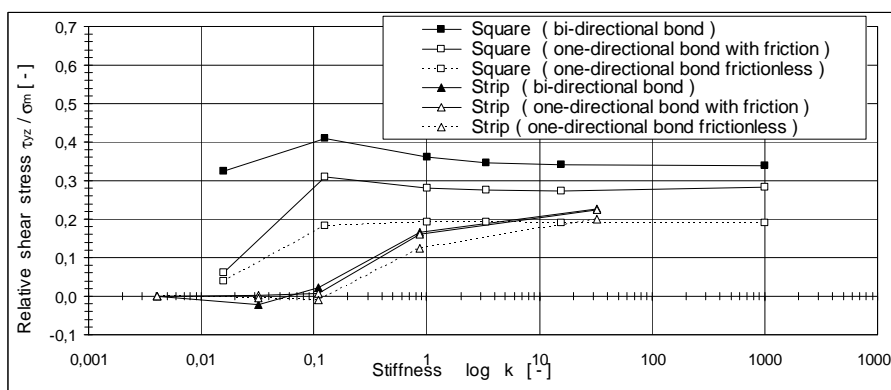


Fig. 5 Effect of stiffness, bond and friction on the values of relative shear contact stresses (τ_{yz} / σ_m) under boundary of foundations

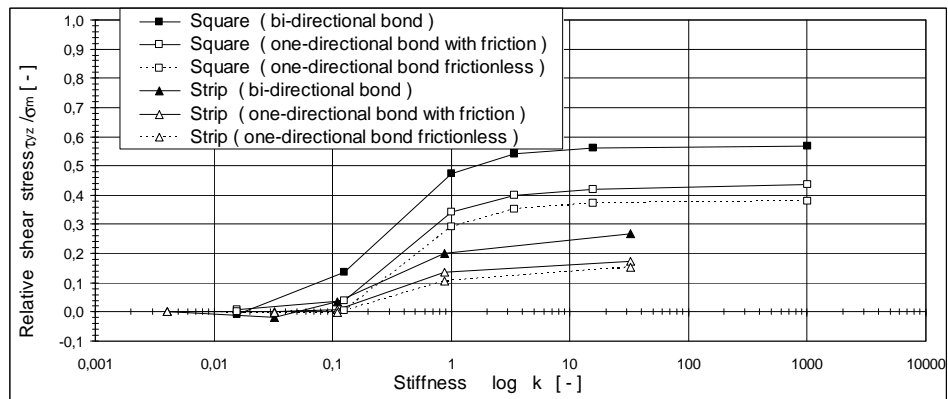


Fig. 6 Effect of stiffness, bond and friction on the values of relative shear contact stresses (τ_{yz}/σ_m) under the corner of foundations

4. Conclusion

Correct assessment of the values of the contact stresses in the foundation bottom affect significantly the reliable design of foundations. Neglecting the effects of the interaction of foundations and subsoil can affect the reliability and economy of foundation building structure. Presented calculations show that taking into account the effect of stiffness, bond and friction in the foundation bottom allows in practical solutions the realistic and economical design of rectangular foundations with respect of the required reliability.

References

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NUMERICKÁ ANALÝZA KONTAKTNÝCH NAPÄTÍ POD PRAVOUHLÝMI (ŠTVORCOVÝM A PÁSOVÝM) PLOŠNÝMI ZÁKLADMI

Zhrnutie

V článku sa zaoberáme numerickou analýzou kontaktných napätí pod tuhými a ohybnými pravouhlými (štvorcovým a pásovým) plošnými základmi. Numerická analýza je zameraná na vplyv tuhosti systému "základová konštrukcia – podložie" a väzby (obojustranná a jednostranná s trením a bez trenia) na veľkosť zvislých normálových a šmykových kontaktných napätí.

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