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SEISMIC ANALYSIS OF RECTANGULAR TANKS – COMPARISON OF MALHOTRA AND HOUSNER MODELS

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1. Introduction

Ground-supported tanks are used to store a variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. Tanks that were inadequately designed or detailed have suffered extensive damage during past earthquakes.

Knowledge of forces, pressures acting on the walls and bottom of containers during an earthquake is important for good design of earthquake resistance structure/facility – tanks, which are made from steel or concrete.

2. Mechanical model of fluid contained in container

The motion of fluid contained in a rigid container may be expressed as the sum of two separate contributions, called "rigid impulsive" and "convective" respectively. The "rigid impulsive" component satisfies exactly the boundary conditions at the walls and the bottom of the tank, but gives (incorrectly, due to the presence of the waves in the dynamic response) zero pressure at the original position of the free surface of fluid in the static situation. The "convective" term does not alter those boundary conditions that are already satisfied, while fulfilling the correct equilibrium condition at the free surface.

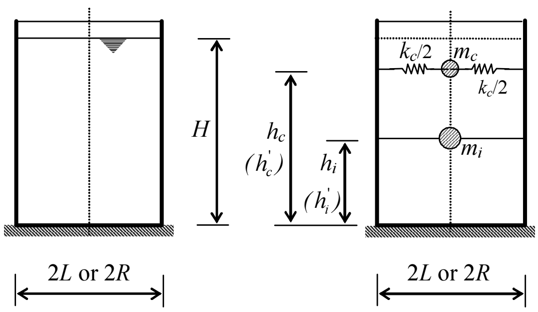

Fig. 1 Spring mass model

When a tank containing liquid vibrates, the liquid exerts impulsive and convective hydrodynamic pressure on the tank wall and the tank base, in addition to the hydrostatic pressure, [1] - [1].

The dynamic analysis of a liquid – filled tank may be carried out using the concept of generalized single – degree – of freedom (SDOF) systems representing the impulsive and convective modes of vibration of the tank – liquid system as shown in Fig. 1. For practical applications, only the first convective mode of vibration needs to be considered in the analysis, mechanical model (Fig. 1). The impulsive mass of liquid mi is rigidly attached to tank wall at height h_i (or $h_i^{'}$). Similarly convective mass m_c is attached to the tank wall at height h_c (or $h_c^{'}$) by a spring of stiffness k_c. The mass, height and natural period of each SDOF system are obtained by the methods described in [2], [4].

Malhotra model. For a ground supported rectangular tank, in which the wall is rigidly connected with the base slab, the natural period of the impulsive mode of vibration $T_i = T_f$ in seconds, is given by [2], [4].

$$T_f = 2 \cdot \pi \cdot \sqrt{d_f / g} \tag{1}$$

where d_f is the deflection of the wall on the vertical centre-line and at the height of the impulse mass, when wall is loaded by a load uniform in the direction of the ground motion and of magnitude $m_i g/(4BH)$.

For a ground supported rectangular, in which the wall is rigidly connected with the base slab, the natural period of the convective mode of vibration $T_c = T_1$, in seconds, is given by

$$T_1 = 2 \cdot \pi \sqrt{L/g \Big/ \frac{\pi}{2} \tanh\left(\frac{\pi}{2} \cdot \frac{H}{L}\right)} \tag{2}$$

Total base shear V of ground supported tank at the bottom of the wall eq. (3), total base shear V' of ground supported tank at the bottom of base slab eq. (4).can be obtained by base shear in impulsive mode and base shear in convective mode,

The overturning moment M of ground supported tank immediately above the base plate is given also by, eq. (5) and the overturning moment M' of ground supported tank immediately below the base plate is given also by, eq. (6).

$$V = (m_i + m_w + m_r)S_e(T_i) + (m_c)S_e(T_c), \tag{3}$$

$$V' = (m_i + m_w + m_b + m_r)S_e(T_i) + (m_c)S_e(T_c), \tag{4}$$

$$M = (m_i h_i + m_w h_w + m_r h_r)S_e(T_i) + (m_c h_c)S_e(T_c), \tag{5}$$

$$M' = (m_i h_i^{'} + m_w h_w + m_b h_b + m_r h_r)S_e(T_i) + (m_c h_c^{'})S_e(T_c), \tag{6}$$

where m_i is the impulsive mass of fluid, m_c - the convective mass of fluid, h_i - height of wall pressure resultant for the impulsive component, h_c - height of wall pressure resultant for the convective component, $h_i^{'}$ - height resultant of pressures on the wall and on the base plate for the impulsive component, $h_c^{'}$ - height resultant of pressures on the wall and on the base plate for the convective component, given by [2], [4]. m_w - mass of the tank wall; m_b - mass of the tank base plate; m_r - mass of the tank roof; h_w - the height of center of gravity of wall mass; h_b - the height of center of gravity of base plate mass; h_r - he height of center of gravity of roof mass. $S_e(T_i)$ is impulsive spectral acceleration, is obtained from a

2% damped elastic response spectrum for steel and prestressed concrete tanks, or a 5% damped elastic response spectrum for concrete and masonry tanks; $S_e(T_c)$ - convective spectral acceleration, is obtained from a 0,5% damped elastic response spectrum. The values of the masses m_i and m_c, as well as of the corresponding heights above the base h_i, h_c, h_i', h_c', calculated for cylindrical tanks and given by [2], [4] for tank's slimness given $\gamma = H/R$, may be adopted for the design of rectangular tanks as well (with L replacing R), with an error less than 15%.

Housner model. We consider ground supported rectangular tank of length L and breadth B, where horizontal earthquake loading acting along length L. The spring mass model for ground supported rectangular tank is based on work of Housner, [1], [5], [6]. Equivalent masses m_i and m_c and heights h_i, h_c, h_i' and h_c' of accelerating liquid can be determined from equations given by [1], [5], [6], depending on H/L ratios, where L is inside length of tank parallel to the direction of seismic force.

Time period of impulsive mode

$$T_i = 2 \cdot \pi \cdot \sqrt{d_f / g}\ , \tag{7}$$

Time period of convective mode of vibration, $T_c = T_1$ in seconds, is given by

$$T_1 = C_c \sqrt{L/g}\ , \tag{8}$$

where C_c is time period for convective mode. Value of C_c can be obtained from [2], [4].

3. Solution, results and discussion

Let us have a ground supported rectangular endlessly long shipping channel, dimension of L = 5 m and height H_w = 3 m. Walls have uniform thickness of 0.25 m. The base slab is h = 0.4 m thick. Shipping channel is filled with water to the height 2.5 m, Fig. 2. There is no roof slab on the tank. The tank is located on hard soil.

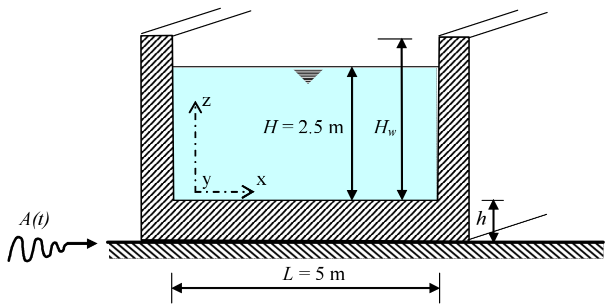

Fig. 2 Details of tank geometry

We consider only horizontal earthquake loading by using of recording of real accelerogram Loma Prieta, California (18.10.1989) along x - direction. We consider with 1 m length of shipping channel.

The comparison of dynamic parameters and shear forces and overturning moments considering equipments by Malhotra used in Eurocode 8 and Housner are in Table 1 and 2.

Table 1. Determined dynamic parameters

Model	m_i [kg]	m_c [kg]	h_i [m]	$h_i^{'}$ [m]	h_c [m]	$h_c^{'}$ [m]	T_i [s]	T_c [s]
by Malhotra	7267	5723	1.094	1.871	1.607	2.032	0.041	2.63
by Housner	7266	6124	0.975	2.000	1.530	2.173	0.041	2.62

Table 2. Base shear *V* and overturning moments *M* and *M'*

Model	*V* [kN]	*M* [kNm]	*M'* [kNm]
by Malhotra	12.81	16.58	28.76
by Housner	12.89	15.83	30.18
Difference %	0.70	4.74	4.94

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Acknowledgements

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Summary

During earthquake activity the liquid in tank exerts impulsive and convective pressures (sloshing). This paper provides comparative study according to models by Malhotra and Housner for calculating of hydrodynamic effect of fluid developed during an earthquake in liquid storage ground-supported rectangular container.