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SEISMIC EFFECT OF LIQUID FILLINGS ON CYLINDRICAL STORAGE TANKS

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1. Introduction

Ground-supported tanks are used to store a variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. The tanks that were inadequately designed have suffered extensive damage during the past earthquakes.

Knowledge of forces, pressures acting on the walls and bottom of containers during an earthquake is important for a proper design of earthquake resistance of structure/facility including tanks which are made from steel or concrete.

2. Mechanical model of fluid contained in container

The motion of fluid contained in a rigid container may be expressed as the sum of two separate contributions, called “rigid impulsive” and “convective” respectively. The “rigid impulsive” component satisfies exactly the boundary conditions at the walls and the bottom of the tank, but gives (incorrectly, due to the presence of the waves in the dynamic response) zero pressure at the original position of the free surface of fluid in the static situation. The “convective” term does not alter those boundary conditions that are already satisfied, while fulfilling the correct equilibrium condition at the free surface.

When a tank containing liquid vibrates, the liquid exerts impulsive and convective hydrodynamic pressure on the tank wall and the tank base in addition to the hydrostatic pressure [1-7].

The dynamic analysis of a ground supported tanks can be idealized as spring-mass model as shown in Fig. 1. The impulsive mass of liquid m_i (near the base of the tank moves with the tank wall) is rigidly attached to tank wall at height h_i (or h_i^*). Similarly convective masses m_c (near the top experiences free-surface sloshing motion) are attached to the tank wall at height h_c (or h_c^*) by a spring of stiffness k_c. The impulsive mass m_i and the convective masses m_c are fractions of the total liquid mass m. Often for practical applications, only the first convective mode of vibration needs to be considered in the analysis of mechanical model. The natural period of vibration of the impulsive mass is from 0.1 [s] to 0.3 [s] and that of the convective mass from 2 [s] to 6 [s].

If one needs to consider additional higher modes of convective masses (m_{cn}), Eqs. 1-8, in which the mass centre of the fluid is referenced may be used:

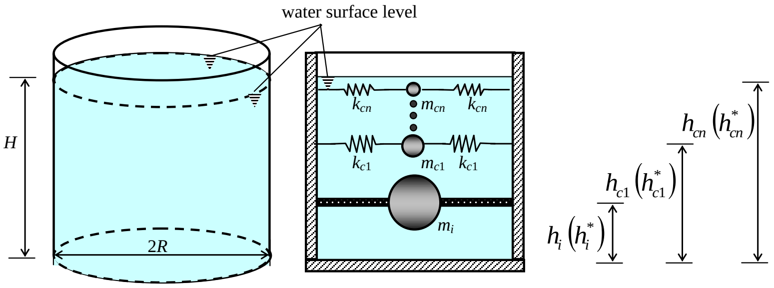

Fig. 1. Spring-mass analogy for ground supported circular tanks.

$$m_i = m2\gamma\sum_{n=0}^{\infty}\frac{I_1(\nu_n/\gamma)}{\nu_n^3 I_1^{'}(\nu_n/\gamma)}\,, \tag{1}$$

$$h_i = H\frac{\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n I_1(\nu_n/\gamma)}{\nu_n^4 I_1^{'}(\nu_n/\gamma)}\left(\nu_n(-1)^n-1\right)}{\displaystyle\sum_{n=0}^{\infty}\frac{I_1(\nu_n/\gamma)}{\nu_n^3 I_1^{'}(\nu_n/\gamma)}}\,, \tag{2}$$

$$h_i^* = H\frac{\dfrac{1}{2}+2\gamma\displaystyle\sum_{n=0}^{\infty}\frac{\nu_n+2(-1)^{n+1}I_1(\nu_n/\gamma)}{\nu_n^4 I_1^{'}(\nu_n/\gamma)}}{2\gamma\displaystyle\sum_{n=0}^{\infty}\frac{I_1(\nu_n/\gamma)}{\nu_n^3 I_1^{'}(\nu_n/\gamma)}}\,, \tag{3}$$

$$m_{cn} = m\frac{2\tanh(\lambda_n H/R)}{(\lambda_n H/R)(\lambda_n^2-1)}\,, \tag{4}$$

$$h_{cn} = H\left(1+\frac{1-\cosh(\lambda_n\cdot H/R)}{(\lambda_n\cdot H/R)\cdot\sinh(\lambda_n\cdot H/R)}\right), \tag{5}$$

$$h_{cn}^* = H\left(1+\frac{2-\cosh(\lambda_n H/R)}{(\lambda_n H/R)\cdot\sinh(\lambda_n H/R)}\right), \tag{6}$$

$$\omega_{cn}^2 = \lambda_n\tanh(\lambda_n H/R)(g/R), \tag{7}$$

$$k_{cn} = m_{cn}\omega_{cn}^2\,, \tag{8}$$

in which: $\nu_n = \dfrac{2n+1}{2}\pi$; $\gamma = H/R$, $I_1(\cdot)$ and $I_1^{'}(\cdot)$ denote the modified Bessel function of order 1 and its derivate and the derivate can be expressed in term of the modified Bessel functions of order 0 and 1 as $I_1^{'}(\cdot)=\dfrac{dI_1(x)}{dx}=I_0(x)-\dfrac{I_1(x)}{x}$; λ_n are the roots of the first-order Bessel function of the first kind (λ_1=1.8412; λ_2=5.3314; λ_3=8.5363, λ_4=11.71 and $\lambda_{5+i}=\lambda_5+5\ i$ (i=1,2,...)); H is the height of fluid filling; h_{cn} and h_{cn}^* are the levels where the oscillators of convective masses need be applied; h_i and h_i^* are the heights of impulsive mass (h_i and h_{cn} are heights of resultant of the impulsive and convective hydrodynamic wall pressure respectively and h_i^* and h_{cn}^* are heights of resultant of the impulsive and convective hydrodynamic wall and bottom pressures respectively); R is the inner radius of container [8].

The total base shearing force V and bending moment M of ground supported tank immediately above the base plate is given by Eqs. 9 and 10 respectively and the overturning moment M^* of ground supported tank immediately below the base plate is given by Eq. 11:

$$V = (m_i + m_w + m_r) S_e(T_i) + (m_c) S_e(T_c), \tag{9}$$

$$M = (m_i h_i + m_w h_w + m_r h_r) S_e(T_i) + (m_c h_c) S_e(T_c), \tag{10}$$

$$M^* = \left(m_i h_i^* + m_w h_w + m_b h_b + m_r h_r\right) S_e(T_i) + \left(m_c h_c^*\right) S_e(T_c), \tag{11}$$

where: m_w is mass of the tank wall; m_b is mass of the tank base plate; m_r is mass of the tank roof; h_w is the height of center of gravity of wall mass; h_b is the height of center of gravity of base plate mass; h_r is the height of center of gravity of roof mass. $S_e(T_i)$ is impulsive spectral acceleration obtained from 2% damped elastic response spectrum for steel and prestressed concrete tanks, or 5% damped elastic response spectrum for concrete and masonry tanks and $S_e(T_c)$ is a convective spectral acceleration obtained from 0,5% damped elastic response spectrum.

3. Example and results

In this study is analyzed fluids filling of ground supported cylindrical rigid tank. The characteristics of fluid filling for $H = 2$ [m], where R is depended on the tank parameter slenderness $\gamma = H/R$. For tank parameter slenderness $\gamma = 0.3$, 0.5, 0.7, 1.2 and 3 are given $R = 6.667$ [m], 4 [m], 2.857 [m], 2 [m], 1 [m] and 0.667 [m]. The material characteristics of fluid filling (H_2O) are: bulk modulus $B = 2.1 \cdot 10^9$ [Pa], density $\rho_w = 1000$ [kg/m^3].

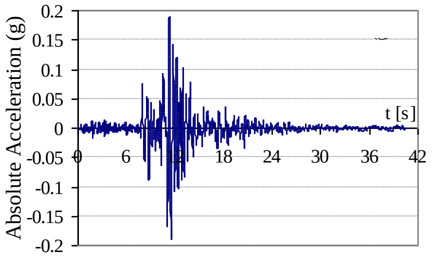

Fig. 2. Accelerogram Loma Prieta, California.

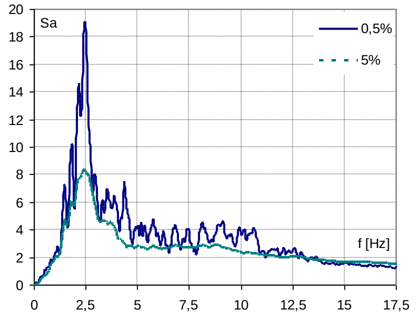

Fig. 3. Elastic response spectrums for accelerogram Loma Prieta from Fig. 2.

As the excitation input we consider a horizontal earthquake load given by the accelerogram of the earthquake in Loma Prieta, California (18.10.1989) shown in Fig. 2. In the analysis we use just the accelerogram for the seismic excitation only in horizontal y - direction. The elastic response spectra of accelerogram Loma Prieta (Fig. 3) were used for the simulation of earthquake. Values of the elastic response spectra were calculated with the damping ratio 5% and 0.5%.

The comparison of shear forces, bending and overturning moments of fluid for different tank slenderness are presented in Fig. 4.

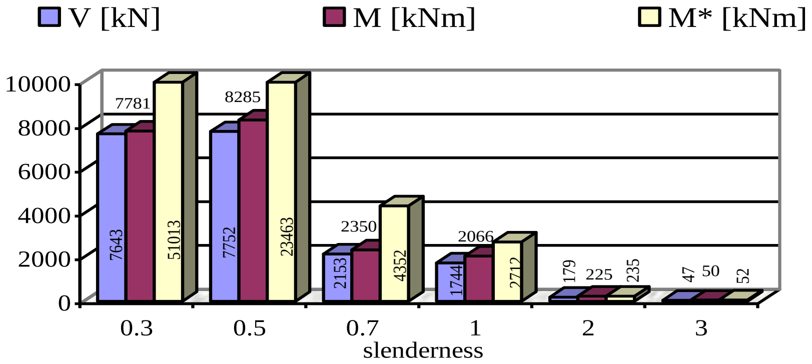

Fig. 4. The total base shears, the bending and overturning moments of fluid for different tank slenderness ratio.

4. Conclusions

Ground-supported cylindrical tanks are used to store a variety of liquids. This paper provides the theoretical background taking into account impulsive and convective (sloshing) actions of the fluid in rigid containers fixed to rigid foundations. Seismic responses are calculated by using response spectra of the earthquake in Loma Prieta, California (18.10.1989). The ground supported rigid cylindrical tanks were analysed in the examples with water filling of height H=2 [m]. The radius of tanks R was depended on the tank slenderness ratio $\gamma=H/R$. The seismic responses: the total base shears, bending and overturning moments, of fluid for different tank slenderness ratio were analysed and compared for the chosen tank slenderness ratios.

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Denotations of symbols

h_{cn}, h_{cn}^* – heights of applying convective masses, [m],
h_i, h_i^* – heights of applying impulsive mass, [m],
h_b – height of center of gravity of base plate mass, [m],
h_r – height of center of gravity of roof mass, [m],
h_w – height of center of gravity of wall mass, [m],
m_b – mass of tank base plate, [kg],
m_c – convective mass of liquid filling, [kg],
m_i – impulsive mass of liquid filling, [kg],
m_r – mass of tank roof, [kg],

m_w – mass of tank wall, [kg],
B – bulk modulus, [Pa],
H – height of liquid filling in tank, [m],
$I_1(\cdot)$ – modified Bessel function,
M – bending moment, [Nm],
M^* – overturning bending moment, [Nm],
R – inner radius of tank, [m],
$S_e(T_c)$ – convective spectral acceleration, [m/s^2],
$S_e(T_i)$ – impulsive spectral acceleration, [m/s^2],
V – base shearing force, [N],
γ – slenderness ratio, [-].

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Summary

Ground-supported cylindrical tanks are used to store a variety of liquids. This paper provides the theoretical background taking into account impulsive and convective (sloshing) actions of the fluid in rigid containers fixed to rigid foundations. Seismic responses were calculated by using response spectra of the earthquake in Loma Prieta, California (18.10.1989). Seismic responses: total base shears, bending and overturning moments of fluid for different tank slenderness ratio were analysed for rigid tanks with water filling of height equal to 2 [m] and radius depended on the tank slenderness ratio.