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POLYMER MATRIX COMPOSITE LAMINATE UNDER HYGROTHERMAL EXPOSURE

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1. Introduction

A laminate is a stack of layers with different fiber orientations, bonded together to attain desired properties. Laminates can be classified as symmetric, asymmetric, balanced and unbalanced composites. In a symmetric laminate, the material and orientation of layers above the mid plane are identical to those below. In a symmetric laminate, the bending-stretching coupling is absent. This is not true for asymmetric laminates.

The analysis of composite structures is more complex and several theories have been proposed for the analysis of laminated composites. Composite laminates have larger planar dimensions, so they can be treated as plate elements. Therefore, plate theories can be applied in the analysis of laminated composites, and it is very important to study the response of these materials to environmental conditions, like temperature and moisture [1].

2. Hygrothermal analysis

The internal forces of a laminate due to hygrothermal conditions in global coordinate system are investigated. The hygrothermal strains in the longitudinal direction and transverse the fiber direction of a lamina are not equal since the effective elastic moduli E_1 and E_2, and also the thermal and moisture expansion coefficients, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, 0)^T$, $\boldsymbol{\beta} = (\beta_1, \beta_2, 0)^T$ respectively, are different.

The stress-strain relations of a unidirectional lamina, including temperature and moisture effects, are given in the following way [2]

$$\boldsymbol{\sigma} = \mathbf{E}\boldsymbol{\varepsilon} \Leftrightarrow \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} (E_{11}) & (E_{12}) & 0 \\ & (E_{22}) & 0 \\ sym. & & (E_{66}) \end{bmatrix} \begin{Bmatrix} \varepsilon_1 - \varepsilon_1^t - \varepsilon_1^h \\ \varepsilon_2 - \varepsilon_2^t - \varepsilon_2^h \\ \gamma_{12} \end{Bmatrix} \tag{1}$$

with

$$\begin{Bmatrix} \varepsilon_1^t \\ \varepsilon_2^t \\ 0 \end{Bmatrix} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \Delta T \qquad \begin{Bmatrix} \varepsilon_1^h \\ \varepsilon_2^h \\ 0 \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} \Delta c$$

where ΔT is the temperature change and Δc is weight of moisture absorption change per unit weight of the lamina, E_{ij} are coefficients of elasticity matrix in local coordinate system.

Note that the temperature and moisture changes do not have any shear strain terms since no shearing is induced in the material axes. One can see that the hygrothermal behaviour of an unidirectional lamina is characterized by two principal coefficients of the thermal expansion and moisture expansion. These coefficients are related to the material properties of fibers and matrix, and of the fiber volume fraction.

The stress-strain relations of a lamina in global coordinate system, including temperature differences, are given as [3,4]

$$\begin{Bmatrix} \sigma_x^t \\ \sigma_y^t \\ \sigma_{xy}^t \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 + z\kappa_x - \alpha_x \Delta T \\ \varepsilon_y^0 + z\kappa_y - \alpha_y \Delta T \\ \gamma_{xy}^0 + z\kappa_{xy} - \alpha_{xy} \Delta T \end{Bmatrix} \tag{2}$$

where $\boldsymbol{\varepsilon}^0$ and $\boldsymbol{\kappa}$ are vectors of strain and curvature of mid-plane, respectively, Q_{ij} are coefficients of elasticity matrix in global coordinate system. The internal forces in the laminate with the influence of temperature are described in the following form

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T \tag{3}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{61} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T \tag{4}$$

In turn, the stress-strain relations of a lamina in global coordinate system, including moisture impact, are given as

$$\begin{Bmatrix} \sigma_{xx}^h \\ \sigma_{yy}^h \\ \sigma_{xy}^h \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 + z\kappa_x - \beta_x \Delta c \\ \varepsilon_y^0 + z\kappa_y - \beta_y \Delta c \\ \gamma_x^0 + z\kappa_{xy} - \beta_{xy} \Delta c \end{Bmatrix} \tag{5}$$

The internal forces in the laminate with the influence of moisture are described in the following form

$$
\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} \Delta c \qquad (6)
$$

$$
\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{61} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} \Delta c \qquad (7)
$$

3. Laminate under mechanical and hygrothermal exposure

The laminates made of 8 layers $[0^o/\theta/-\theta/90^o]_s$ and $[0^o/\theta/-\theta/90^o/0^o/\theta/-\theta/90^o]$ were considered. Each carbon/epoxy composite layer has the same material characteristics: $E_1 = 138$ GPa, $E_2 = 10.32$ GPa, $G_{12} = 4.39$ GPa, $\nu_{12} = 0.34$, $\alpha_1 = -3.10^{-7}$ m/(m·°C), $\alpha_1 = 2.8^{-5}$ m/(m·°C), $\beta_1 = 0.01$ m/m, $\beta_2 = 0.3$ m/m, $X_t = 2000$ MPa, $X_c = 1200$ MPa, $Y_t = 50$ MPa, $Y_c = 170$ MPa, $S = 70$ MPa. The temperature change $\Delta T = 20$°C and moisture concentration change $\Delta c = 1.5$ %. The reinforcement angle θ is changing from 0° to 90°. The laminate plate is under mechanical loading – uniform distributed pressure $q = 1.4$ kPa. The results of mechanical and hygrothermal conditions are solved for simply supported square laminate plate with $L = 0.8$ m and thickness $h = 0.008$ m.

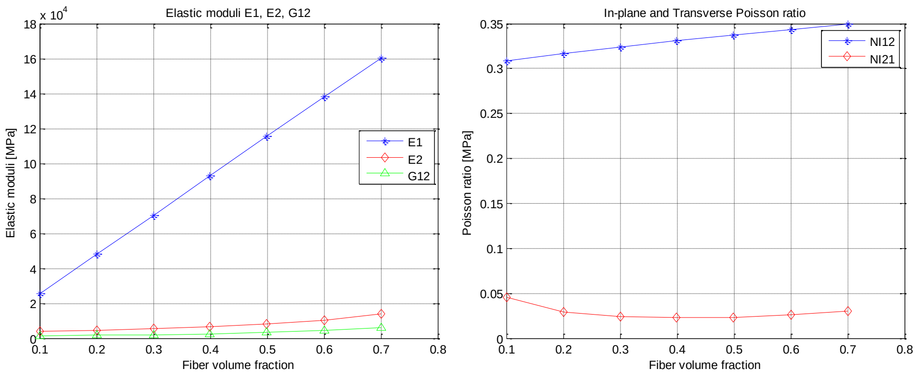

Fig. 1. Elastic properties versus fiber volume fraction.

The material characteristics of laminate layer versus fiber volume fraction can be seen in Fig. 1. In Fig. 2, there is drawn the influence of fiber reinforced angle change on strength ratio. A more general form of the failure criterion for orthotropic materials is expressed as

$$F_{01}\sigma_1 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{02}\sigma_2 + F_{22}\sigma_2^2 + F_{44}\tau_{12}^2 < 1 \tag{8}$$

where

$$F_{01} = \frac{1}{X_t} - \frac{1}{X_c}, \quad F_{11} = \frac{1}{X_t X_c}, \quad F_{02} = \frac{1}{Y_t} - \frac{1}{Y_c},$$

$$F_{22} = \frac{1}{Y_t Y_c}, \quad F_{12} = -\frac{1}{2}\frac{1}{\sqrt{X_t X_c Y_t Y_c}}, \quad F_{44} = \frac{1}{S^2}$$

It can be seen from Fig. 2, the best laminate is characterised by fiber reinforced angle θ = 45°.

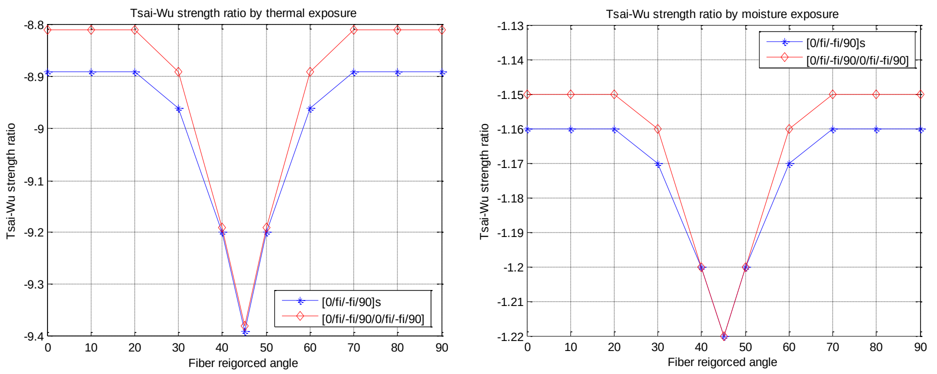

Fig. 2. Thermal and moisture strength of ratio versus fiber reinforced angle

Table 1. Bending moments of mechanical and hygrothermal exposure for $[0^o/45^o/{-45}^o/90^o]_s$

Property	M_x [MNm/m]	M_y [MNm/m]	M_x [MNm/m]
q	$6.15 \cdot 10^{-5}$	$2.33 \cdot 10^{-5}$	$5.41 \cdot 10^{-5}$
Δc	0	0	0
ΔT	0	0	0

Table 2. Normal forces of mechanical and hygrothermal exposure for $[0^o/45^o/{-45}^o/90^o]_s$

Property	$N_x = N_y$ [MN/m]	N_{xy} [MN/m]
q	0	0
Δc	$3.37 \cdot 10^{-1}$	0
ΔT	$2.79 \cdot 10^{-2}$	0

Table 3. Bending moments of mechanical and hygrothermal exposure for [0º /45º /–45º /90º /0º /45º /–45º /90º]

Property	M_x [MNm/m]	M_y [MNm/m]	M_{xy} [MNm/m]
q	$1.2 \cdot 10^{-5}$	$2.54 \cdot 10^{-5}$	$-6.29 \cdot 10^{-6}$
Δc	$3.1 \cdot 10^{-5}$	$-3.1 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$
ΔT	$1.39 \cdot 10^{-5}$	$-1.39 \cdot 10^{-5}$	$0.461 \cdot 10^{-5}$

Table 4. Normal forces of mechanical and hygrothermal exposure for [0º /45º /–45º /90º /0º /45º /–45º /90º]

Property	N_x [MN/m]	N_y [MN/m]	N_{xy} [MN/m]
q	$2.84 \cdot 10^{-2}$	$3.22 \cdot 10^{-2}$	$1.26 \cdot 10^{-3}$
Δc	$3.37 \cdot 10^{-1}$	$3.37 \cdot 10^{-1}$	0
ΔT	$2.79 \cdot 10^{-2}$	$2.79 \cdot 10^{-2}$	0

4. Conclusion

There were investigated the laminates made of 8 layers $[0^o/\theta/-\theta/90^o]_s$ and $[0^o/\theta/-\theta/90^o/0^o/\theta/-\theta/90^o]$ in the paper. Each layer consists of carbon fibers and Epoxy matrix with fiber volume fraction $\xi = 0.6$. The material characteristics of one laminate layer were establish using the program that works under Mori-Tanaka method [2] (Fig. 1). Then, there was calculated the influence of fiber reinforced angle change on thermal and moisture strength ratio of laminates (Fig. 2). It is seen the best laminates are $[0^o/45^o/-45^o/90^o]_s$ and [0º /45º /–45º /90º /0º /45º /–45º /90º]. The response of laminate composite to hygrothermal conditions was investigated to validate the hygrothermal aspect of the model using the program MATLAB. The internal forces without degradation of material are written in the Tables 1-4. The symmetric laminate (matrix $\mathbf{B} = \mathbf{0}$) has not any coupling effect between normal forces and bending moments (Tables 1 and 2), opposite the unsymmetric laminate is characterised by the coupling effect (Tables 3 and 4).

Based on these results, a designer can choose the right ply orientations to control behaviour of laminated plates [5,6]. It is possible to minimize the environmental effect by judiciously selecting the laminate configuration. Consequently, it is preferable to use the symmetric laminate panels for exterior utilization in buildings.

Denotations of symbols

$\boldsymbol{\sigma}$ - stress tensor, [Pa],
$\mathbf{E}$ - elasticity tensor, [Pa],
$\boldsymbol{\varepsilon}$ - strain tensor, [-],
E_1, E_2 - modulus of elasticity in longitudinal and transversal direction, [Pa],
ν_{12} - in-plane Poisson ratio, [-],
G_{12} - in-plane shear modulus, [Pa],
E_{ij} - coefficients of elasticity matrix in local coordinate system, [Pa],
Q_{ij} - coefficients of elasticity matrix in global coordinate system, [Pa],
ΔT - temperature change, [ºC],

Δc - weight of moisture absorption change per unit weight of the lamina, [%],
$\boldsymbol{\varepsilon}^0$ - vector of strain of mid-plane, [-],
$\boldsymbol{\kappa}$ - vector of curvature of mid-plane, [1/m],
α_1, α_2 - coefficients of thermal expansion in longitudinal and transversal direction, [m/(m·°C)],
β_1, β_2 - coefficient of moisture expansion in longitudinal and transversal direction, [m/m],
A - in-plane stiffness matrix, [N],
B - coupling stiffness matrix, [N·m],
D - bending stiffness matrix, [N·m^2],
X_t, X_c - tensile and compressive strength in longitudinal direction, [Pa],
Y_t, Y_c - tensile and compressive strength in transversal direction, [Pa],
S - shear strength, [Pa].

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Summary

The static load model of laminate with the hygrothermal load is considered in the paper. The corresponding ply properties are calculated using Mori-Tanaka method. A parametric study is conducted by varying the fiber volume fraction in one laminate layer and the fiber orientation of the angle plies in the laminate. The internal forces of laminate plate with symmetric and unsymmetric stacking of layers are investigated.