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MODELLING IN NON-LINEAR VISCOELASTICITY

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1. Introduction

Considering the difficulty of descriptions of long-lasting deformation of body analyzed in consequence of significant influence of nonlinear components in time-deformation, the large efforts for derivation of stress-strain dependences were exerted [1,2]. Despite the great effort made, solving this problem for generally valid mathematical model was unsuccesful. Diversity of mechanical response of materials under large levels of internal stresses makes this problem even more complicated. The difference in time behavior of silicate composite materials compared with metals under high temperature and with synthetic material on basis of polyvinylchloride is well known phenomenon.

2. Assumptions of the solution

The present non-linear phenomenological creep theory of ageing body is starting from the following hypotheses:

- the body is homogeneous, isotropic and continuously deformable
- total deformations of body arising from the long term loading consists from the instantaneous elastic component ε_{pr}, linear ε_{ln} and non-linear components of creep ε_{nl}
- for the total linear deformation (instantaneous elastic deformation and linear creep component) the principle of superposition is valid

Further, we assume, th125
at for $t < \tau_1$ (time of the initial acting of load) internal stresses are negligible. Then for overall deformation in the time $t \geq \tau_1$ follows

$$\varepsilon^*(t) = \varepsilon_{pr}(t) + \varepsilon_{ln}(t) + \varepsilon_{nl}(t) \tag{1}$$

3. Mathematical models

On the basis of above-mentioned assumptions and due to existence of linear components of deformations we may express Eq (1) in other form [1, 2]

$$\varepsilon^*(t) = \frac{\sigma(t)}{E(t)} - \int_{\tau_1}^{t} \sigma(t) \frac{\partial \delta(t,\tau)}{\partial \tau} d\tau + \varepsilon_{nl}(t) \tag{2}$$

where $\delta(t,\tau) = \frac{1}{E(\tau)} + C_{ln}(t,\tau)$ is linear part of specific deformation and $C_{ln}(t,\tau)$ is contracted degree of creep.

From the experimental measuring of creep curves for concrete [3,4] the finite conditions $(t \to \infty)$ are valid

$$t \to t^*,\ \varepsilon_r \to \varepsilon_r{}^*,\ \dot{\varepsilon}_r \to \dot{\varepsilon}^*(\dot{\varepsilon}_r \to \infty) \tag{3}$$

$$t \to \infty,\ \eta = \eta^*,\ \varepsilon_r \to \varepsilon_r{}^*,\ \dot{\varepsilon}_r \to const,\ (\varepsilon_r \to \infty) \tag{4}$$

where $\eta = \sigma / R$.

Modeling of deformation component $\varepsilon_{nl}(t)$ may cause some difficulties. By its derivation a great deal of specific long-lasting deformation of body material, e.g. influence of dead or fluctuating components of stresses, level of stress, age of material in time of loading, mode and values of components of previous stresses, effects of environment (humidity, temperature) etc.

4. Non-linear hereditary theory of ageing

Additional assumption for version of non-linear theory is a special principle of superposition which for $\sigma(\tau_1) = const$ has a form

$$\varepsilon_{nl}(\sigma; t, \tau_1) = \mathcal{F}_1[\sigma(\tau_1)] C_{nl}(t,\tau) \tag{5}$$

where $\mathcal{F}_1[\sigma(\tau_1)]$ is a stress function fulfilling certain properties e.g. [6]. In the case of stepwise increasing stresses $\Delta\sigma(\tau_1), \Delta\sigma(\tau_2),...$ it is possible to determine the deformation component $\varepsilon_{nl}(t)$ on the basis of curves set $C_{nl}(t,\tau_i)$ similarly as in linear theory, that is

$$\varepsilon_{nl}(t) = \sum_{i=1}^{n} \mathcal{F}_1[\Delta\sigma(\tau_i)] C_{nl}(t,\tau_i) \quad t > \tau_n \tag{6}$$

In the case of continuous course of stresses, it is possible to generalize relation (6) as follows

$$\varepsilon_{nl}(t) = \mathcal{F}_1[\sigma(\tau_1)] C_{nl}(t,\tau_1) + \int_{\tau_1}^{t} \frac{d\mathcal{F}_1[\sigma(\tau)]}{d\tau} C_{nl}(t,\tau) d\tau \tag{7}$$

and after integration per partes we have

$$\varepsilon_{nl}(t) = -\int_{\tau_1}^{t} \mathcal{F}_1[\sigma(\tau)] \frac{\partial C_{nl}(t,\tau)}{\partial \tau} d\tau \tag{8}$$

The resulting deformation in time $t > \tau_1$ we may determinate in the form

$$\varepsilon^*(t) = \frac{\sigma(t)}{E(t)} - \int_{\tau_1}^{t} \sigma(\tau) \frac{\partial \delta(t,\tau)}{\partial \tau} d\tau - \int_{\tau_1}^{t} \mathcal{F}_1[\sigma(\tau)] \frac{\partial C_{nl}(t,\tau)}{\partial \tau} d\tau \tag{9}$$

where $\mathcal{F}_1(\sigma) = \sigma f_{nl}(\sigma) = \sigma \sum_{i=1}^{h} b_i \sigma^i$ (b_i are constants obtained from the experimental measuring), for example, for intensive ageing of concrete (τ_1<28days), $C(t,\tau) \neq C(t-\tau)$, $E \neq const$ Alexandrovskij and Popkovová [3] recommended relations

$$C(t,\tau) = \theta(\tau) - \psi(t) \frac{e^{\gamma\tau} - B}{e^{\gamma t} - B} - \Delta(\tau)\partial(t-\tau) \tag{10}$$

where $\theta(\tau) = \psi(\tau) + \Delta(\tau)$, $\lambda(0) = 1$, $\lambda(\infty) = 0$, $E(\tau) = E_o[1 - \beta_1 e^{-\alpha_1 \tau} - \beta_2 e^{-\alpha_2 \tau}]$, $\beta_1 \in \langle 0,1 \rangle$, $\beta_2 \in \langle 0,1 \rangle$ and $\beta_1 + \beta_2 \leq 1$. These relations represent the initial process of hardening of concrete. On the assumption that $\sigma(\tau_1) = const$ and the superposition principle for non-linear component $\varepsilon_{nl}(t)$ is valid, it is possible to determine the physical equation for $\varepsilon^*(t)$ in the form

$$\varepsilon^*(t) = \frac{\sigma(t)}{E(t)} - \int_{\tau_1}^{t} \sigma(\tau) \frac{\partial}{\partial \tau} \left[\frac{1}{E(\tau)} \right] d\tau - \int_{\tau_1}^{t} \mathcal{F}[\sigma(t)] \frac{\partial C(t,\tau)}{\partial \tau} d\tau \tag{11}$$

The stress function $\mathcal{F}(\sigma) = \sigma f(\sigma)$ can be approximated as follows

$$\mathcal{F}(\sigma) = b \left(\frac{\sigma}{R} \right)^m, \quad \mathcal{F}(\sigma) = a + b \left(\frac{\sigma}{R} \right)^m$$

where a and b are material's constants. The selection of functions $\mathcal{F}[\sigma(t)]$ is rather complicated problem and it depends on the stress level, in the moment of loading the structure. E.g. according to [1] the following approximation is used

$$\mathcal{F}[\sigma(t)] = \left[1 + \beta_{10}\sigma(\tau) + \beta_{11}\sigma^2(\tau) + \beta_{12}\sigma^3(\tau) \right] \sigma(\tau) \tag{12}$$

The relations (9) and (11) were derived on the basis of the superposition principle for linear and nonlinear components of creep deformations and for the design practice are very effective. Of course this procedure has an own limit of application. Complicated mode of loading changes or factor of high level of stresses cause differences in the experimental and theoretical results. What are further generalized solutions? For example in [4] the relation (11) for the case of instentaneous elastic deformation is generalized and has form

$$\varepsilon^*(t) = \frac{\sigma^*(t)}{E(t)} F_M \left[\sigma^*(t) \right] - \int_{\tau_1}^{t} \sigma^*(\tau) F_n \left[\sigma^*(\tau) \right] \frac{\partial}{\partial \tau} C^*(t,\tau) d\tau \tag{13}$$

where $\mathcal{F}_j[\sigma^*(t)]$, $j=M,n$ are non-linear stress-functions and $C^*(t,\tau)d\tau$ is a creep degree expressed in a form

$$C^*(t,\tau) = C(t,\tau) + \frac{1}{E(\tau)} - \frac{1}{E(t)} \tag{14}$$

In order to make better approximation of curves describing the long-term deformation process under high levels of stresses the „three-stage“ equation is suggested

$$\varepsilon^*(t)E(t) = \sigma(t) + \int_{\tau_1}^{t} \sigma(\tau)\mathcal{K}_1(t,\tau)d\tau + \int_{\tau_1}^{t} \mathcal{F}_1[\sigma(\tau)]\mathcal{K}_2(t,\tau)d\tau + \int_{\tau_1}^{t} \mathcal{F}_2[\sigma(\tau)]\mathcal{K}_3(t,\tau)d\tau \tag{15}$$

where second term represents the linear creep, third one represents the component of non-linear creep and fourth term represents the non-fading component of linear creep.

Notation

R − a stress in the progressive growth of microcracks,
ε_r − deformation in creep in the moment of failure of specimen material,
t − a time of loading before failure of speciment;

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Summary

We may see that the modeling of viscoelastic response on the effects of external loading of structures is rather complicated, provided by suitable selection of set of the parameters for determination the stress-function $\mathcal{F}[\sigma(t)]$. A given case one-dimensional problem was solved. Different method for approximation of solution of the given problem is represented by the so-called geometrical principle of homothety of creep curves and isochrone curves. Presented process of problem solution from the mathematical point of view is not demanding and thus it is widely used in practice.